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SOLUTION BY THE PROPOSER.

As this triangle is designed to show the slope of earth embankments the word 'slope' is defined as it is understood by the civil engineer: the ratio of horizontal to vertical distance.

Let AD, BE, CF be the medians of the required triangle, ABC , then

$$(1) \quad 2 \cot ADB = \cot C - \cot B,$$

$$(2) \quad 2 \cot BEC = \cot A - \cot C,$$

$$(3) \quad 2 \cot CFA = \cot B - \cot A,$$

and hence $\cot ADB + \cot BEC + \cot CFA = 0$. If we take $\cot ADB = \frac{2}{3}$ then from the assigned values: (a) $\cot BEC = -1$, $\cot CFA = \frac{1}{3}$; (b) $\cot BEC = \frac{1}{3}$, $\cot CFA = -1$. Considering case (a), these values inserted in (1), (2), (3) give two independent equations, which are to be combined with the identity,

$$(4) \quad \cot B \cot C + \cot C \cot A + \cot A \cot B = 1.$$

Putting, from (2) and (3), $\cot B = \cot A + \frac{2}{3}$, $\cot C = \cot A + 2$, equation (4) becomes

$$9 \cot^2 A + 16 \cot A + 1 = 0;$$

whence,

$$\cot A = \frac{-8 \pm \sqrt{55}}{9}; \quad \cot B = \frac{-2 \pm \sqrt{55}}{9}; \quad \text{and} \quad \cot C = \frac{10 \pm \sqrt{55}}{9}.$$

Since a triangle can have only one obtuse angle only one cotangent can be negative. Consequently, the (−) before the radical does not lead to a solution. Using the (+) sign,

$$A = 93^\circ 42' 42'', \quad B = 58^\circ 57' 37'', \quad C = 27^\circ 19' 41''.$$

When $c = 10.00$, $a = 21.74$ and $b = 18.66$.

Case (b) can be treated in the same way and we get

$$\cot A = \frac{8 + \sqrt{55}}{9}, \quad \cot B = \frac{-10 + \sqrt{55}}{9}, \quad \cot C = \frac{2 + \sqrt{55}}{9}.$$

2745 [1919, 37]. Proposed by G. I. HOPKINS, Manchester, N. H.

A recent English publication contains the following method of constructing a regular polygon of 17 sides: Draw the radius CB perpendicular to the diameter AQ of the circle whose center is B . On BC lay off BD equal to one-fourth of BC . On BA , lay off BE and draw DE making angle BDE one fourth of angle BDA . On BQ lay off BF and draw DF , making angle FDE 45° . On AF as diameter, construct semi-circle FHA intersecting CB in H . With E as center and EH as radius construct semi-circle LHK intersecting CB in H . At the points L and K draw the ordinates NL and MK . Bisect the arc NM and let P be the point of bisection. Then the chord NP ($= MP$) is a side of the regular polygon of 17 sides. Is the method of construction correct?

I. SOLUTION BY C. H. CHEPPELL, Hove, England.

The abscissas BK , BL , and their ordinates to M and N , make the angles MBA , NBA equal to $10\pi/17$, and $6\pi/17$ respectively. Consequently the difference of these two angles, the angle MBN , is equal to $4\pi/17$. This justifies the claim of the construction.

But tables of all the trigonometrical functions of $n\pi/17$ are not readily available; and it may be more satisfactory if we outline the connection between the numerical value of NM and the numerical value of some function given in a standard work.

Taking the radius of the circle ACQ as unity, we find

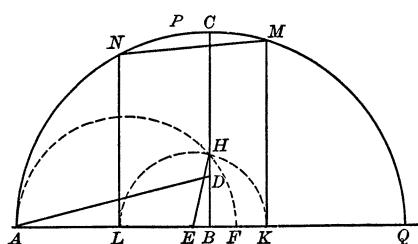
$$BE = \frac{-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}}{16},$$

$$BF = \frac{+1 - \sqrt{17} + \sqrt{34 - 2\sqrt{17}}}{16},$$

$$EH = \frac{\sqrt{2(\sqrt{17} - 3)(2\sqrt{17} + \sqrt{34 + 2\sqrt{17}})}}{16} = \frac{\beta}{16},$$

$$AK = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} + \beta}{16},$$

$$AL = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} - \beta}{16},$$



$$KQ = \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} - \beta}{16},$$

$$LQ = \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} + \beta}{16},$$

$$LN = \sqrt{AL \cdot LQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} - 2\beta'},$$

$$KM = \sqrt{AK \cdot KQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} + 2\beta'},$$

$$[\beta' = \sqrt{2(\sqrt{17} - 3)(2\sqrt{17} - \sqrt{34 + 2\sqrt{17}})}; \text{ and } \beta \times (-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}) = 4 \cdot \beta'],$$

$$LN \times KM = \frac{1}{64} \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} + \sqrt{34 - 2\sqrt{17}})} = \frac{4\alpha}{64},$$

$$NM^2 = LK^2 + (KM - LN)^2$$

$$= 4 \cdot EH^2 + KM^2 + LN^2 - \frac{8\alpha}{64}$$

$$= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 8\alpha}{64},$$

$$[\alpha' = \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}; \text{ and } 4\alpha = \alpha' \times (-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})],$$

$$\therefore NM^2 = \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64}.$$

And, in the circle with radius equal to unity, NM represents the value $2 \sin (2\pi/17) \times 1$; and therefore

$$\begin{aligned} 4 \cos^2 \frac{2\pi}{17} &= 4 - NM^2 \\ &= \frac{+120 + 8\sqrt{17} - 8\sqrt{34 - 2\sqrt{17}} + 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64} \\ &= \left[\frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \alpha'}{8} \right]^2 \end{aligned}$$

and

$$2 \cos \frac{2\pi}{17} = \frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}}{8}.$$

And this value of $2 \cos (2\pi/17)$ will be found to agree with that given in Klein's *Famous Problems in Elementary Geometry* (Beman & Smith), though our α' is there written in a different form.

II. HISTORICAL NOTE BY R. C. ARCHIBALD, Brown University.

This method of construction is due to H. W. Richmond, *Quarterly Journal of Mathematics*, Volume 26, 1893, pp. 206-207; and *Mathematische Annalen*, Volume 67, 1909, pp. 460-461. It is reproduced on page 34 of H. P. Hudson's *Ruler and Compasses*, London, 1916.

Various constructions of the regular polygon of seventeen sides were reviewed by R. Goldenring in his *Die elementargeometrischen Konstruktionen des regelmässigen Siebzehnecks* (Leipzig, 1915), but many omissions in this professedly complete survey were noted by the writer in the *Bulletin of the American Mathematical Society*, vol. 22, 239-246. The first solution in an English publication, given by Lowry in 1819,¹ was reproduced in this *Monthly*² in 1899 and 1914. Other solutions and historical notes are set forth in the articles printed above, pages 322-326.

2767 [1919, 171]. Proposed by W. W. JOHNSON, U. S. Naval Academy.

Let the complex quantities p , q , and r satisfy the relation $p^2 + q^2 + r^2 = 0$; prove that the corresponding vectors OP , OQ , and OR are such that if any two of them are taken as conjugate semi-diameters of an ellipse, the third lies on the minor axis, and its length is the distance from the center to either focus.

SOLUTION BY A. PELLETIER, Montreal, Can.

Let $(x^2/a^2) + (y^2/b^2) = 1$, be the equation of the ellipse having OP and OQ for conjugate semi-diameters ($2a$ and $2b$ being the axes, and $a \geq b$). If α , α' , α'' are the respective arguments of

¹ *The Mathematical Repository*, new series, vol. 4, p. 160; Lowry's proof occupies pages 160-168.

² Volume 6, p. 239 and volume 21, p. 252.